Department of Physics
Problems for Individual Home Tasks. 3rd year, Spring Term

Solve time-independent Schrödinger Equation for one-dimensional movement in $U(x)$

1. Find energy levels with E≥ 0 and wave functions for $U(x)=\begin{cases}\infty, |x|\ge a;\\ -\alpha\delta(x), |x|\le a.\end{cases}$

2. Find energy levels with E<0 and wave functions for $U(x)=\begin{cases}\infty, |x|\ge a;\\ -\alpha\delta(x), |x|\le a.\end{cases}$.

3. Find wave functions of stationary states with $E > 0$ for $U(x)=\begin{cases}\infty, x\le 0;\\ -\alpha\delta(x-a), x>0.\end{cases}$

4. Find wave functions of stationary states with $E > 0$ for $U(x)=\begin{cases}-\alpha\delta(x), x\le a;\\ \infty, x>a.\end{cases}$

5. Find energy levels with E< 0 and wave functions for $U(x)=\begin{cases}U_0, |x|\ge a;\\ -\alpha\delta(x), \ |x|\le a.\end{cases}$

6. Find energy levels with E≥ 0 and wave functions for $U(x)=\begin{cases}\infty, x\le 0, \ge 2a;\\ -\alpha\delta(x-a), 0<x<2a.\end{cases}$

7. Find energy levels with E<0 and wave functions for $U(x)=\begin{cases}\infty, x\le 0, \ge 2a;\\ -\alpha\delta(x-a), 0<x<2a.\end{cases}$

8. Find energy levels with E<0 and wave functions for $U(x)=-\alpha\big(\delta(x+b)+\delta(x-a)\big)$.

9. Find the transmission coefficient for over-barrier scattering for two delta-wells $U(x)=-\alpha\big(\delta(x+a)+\delta(x-a)\big)$.

10. Find the reflection coefficient for over-barrier scattering for two delta-wells $U(x)=-\alpha\big(\delta(x+a)+\delta(x-a)\big)$.

11. Find energy levels with E<0 and wave functions for $U(x)=-\alpha\big(\delta(x)+\delta(x-a)\big)$.

12. Find the transmission coefficient for over-barrier scattering for two delta-wells $U(x)=-\alpha\big(\delta(x)+\delta(x-a)\big)$.

13. Find wave functions of stationary states with $E > 0$ for $U(x)=\begin{cases}\alpha\delta(x), x\le a;\\ U_0, x>a.\end{cases}$

14. Find wave functions of stationary states with $E > 0$ for $U(x) = \begin{cases} U_0, x \le 0; \\ \alpha\delta(x-a), x > 0. \end{cases}$

15. Find wave functions of stationary states with $E > 0$ for $U(x) = \begin{cases} \alpha\delta(x), x \le a; \\ \infty, x > a. \end{cases}$

16. Find energy levels with $0 < E \le U_0$ for $U(x) = \begin{cases} U_0, |x| \ge a; \\ \alpha\delta(x), \ |x| \le a. \end{cases}$

17. Find wave functions of stationary states for $U(x) = \begin{cases} \infty, x \le 0; \\ \alpha\delta(x-a), x > 0; \end{cases}$

18. Find energy levels with E< 0 and wave functions for $U(x) = \alpha\left(\delta(x+a) - \delta(x-a)\right).$

19. Find energy levels with E< 0 and wave functions for $U(x) = \alpha\left(-\delta(x+a) + \delta(x-a)\right).$

20. Find energy levels with E< 0 and wave functions for $U(x) = \alpha\left(\delta(x) - \delta(x-a)\right).$

21. Find energy levels with E< 0 and wave functions for $U(x) = \alpha\left(-\delta(x) + \delta(x-a)\right).$

22. Find the reflection coefficient for $U(x) = \alpha\left(\delta(x) + \delta(x-a)\right).$

23. Find the reflection coefficient for $U(x) = \alpha\left(\delta(x+a) + \delta(x-a)\right).$

24. Find the transmission coefficient for $U(x) = \alpha\left(-\delta(x) + \delta(x-a)\right).$

25. Find wave functions of stationary states with E < U_2 < U_1 for $U(x) = \begin{cases} 0, x \le 0; \\ U_1, 0 < x \le a; \\ U_2, x \ge a. \end{cases}$ What is the transmission coefficient in this case?

26. Find wave functions of stationary states with E < U_1 < U_2 for $U(x) = \begin{cases} 0, x \le 0; \\ U_1, 0 < x \le a \\ U_2, x \ge a \end{cases}$. What is the transmission coefficient in this case?

27. Find the transmission coefficient for $E > U_1 > U_2$ in $U(x) = \begin{cases} 0, x \le 0; \\ U_1, 0 < x \le a; \\ U_2, x \ge a. \end{cases}$.

28. Find the transmission coefficient for $U_2 < E < U_1$ in $U(x) = \begin{cases} 0, x \le 0; \\ U_1, 0 < x \le a; \\ U_2, x \ge a. \end{cases}$

29. Find the over-barrier transmission coefficient for $U(x) = \begin{cases} 0, x \le 0; \\ U_1, 0 < x \le a; \\ U_2, x \ge a. \end{cases}$ at $E > U_2 > U_1$

30. Find the over-barrier transmission coefficient for $U(x) = \begin{cases} U_1, x \le 0; \\ 0, 0 < x \le a; \\ U_2, x \ge a. \end{cases}$ at $E > U_2 >. U_1.$

31. Find bound states in $U(x) = \begin{cases} \infty, x \le 0; \\ -U_0, 0 < x \le a; . \\ 0, x \ge a. \end{cases}$

32. Find wave functions of stationary states with $E > 0$ for $U(x) = \begin{cases} \infty, x \le 0; \\ -U_0, 0 < x \le a. \\ 0, x \ge a \end{cases}$

33. Find the transmission coefficient for $E > U_1 > U_2$ $U(x) = \begin{cases} 0, x \le 0; \\ U_1, 0 < x \le a; \\ U_2, x \ge a. \end{cases}$

34. Find bound states and wave functions (WF) for $0 < E \le U_0$ in $U(x) = \begin{cases} U_0, \lvert x \rvert \le b; \\ 0, b \le \lvert x \rvert \le a; \\ \infty, \lvert x \rvert \ge a. \end{cases}$

Consider only case of odd WF.

35. Find bound states and wave functions (WF) for $E > U_0$ in $U(x) = \begin{cases} U_0 \ , |x| \le b; \\ 0, \ b \le |x| \le a; \\ \infty \ , |x| \ge a. \end{cases}$ Consider only case of odd WF.

36. Find bound states and wave functions (WF) for $0 < E \le U_0$ in $U(x) = \begin{cases} -U_0 \ , |x| \le b; \\ 0, \ b \le |x| \le a; \\ \infty \ , |x| \ge a. \end{cases}$ Consider only case of odd WF.

37. Find bound states and wave functions (WF) for $E > U_0$ in $U(x) = \begin{cases} -U_0 \ , |x| \le b; \\ 0, \ b \le |x| \le a; \\ \infty \ , |x| \ge a. \end{cases}$ Consider only case of odd WF.

38. Find bound states and wave functions (WF) for $0 < E \le U_0$ in $U(x) = \begin{cases} U_0, |x| \le b; \\ 0, b \le |x| \le a; \\ \infty, |x| \ge a. \end{cases}$ Consider only case of even WF.

39. Find bound states and wave functions (WF) for $E > U_0$ in $U(x) = \begin{cases} U_0 \ , |x| \le b; \\ 0, \ b \le |x| \le a; \\ \infty \ , |x| \ge a; \end{cases}$ Consider only case of even WF.

40. Find bound states and wave functions (WF) for $0 < E \le U_0$ in $U(x) = \begin{cases} -U_0 \ , |x| \le b; \\ 0, \ b \le |x| \le a; \\ \infty \ , |x| \ge a. \end{cases}$ Consider only case of even WF.

41. Find bound states and wave functions (WF) for $E > U_0$ in $U(x) = \begin{cases} -U_0 \ , |x| \le b; \\ 0, \ b \le |x| \le a; \\ \infty \ , |x| \ge a; \end{cases}$ Consider only case of even WF.

42. Find bound states and wave functions (WF) for $0 < E \le U_0$ in $U(x) = \begin{cases} \infty, x \le 0; x \ge a; \\ U_0, \ x \le b; \\ 0, b < x < a. \end{cases}$

43. Find bound states and wave functions (WF) for $E > U_0$ in $U(x) = \begin{cases} \infty, x \le 0; x \ge a; \\ U_0, \ x \le b; \\ 0, b < x < a. \end{cases}$

44. Find bound states and wave functions (WF) for $-U_0 < E \le 0$ in $U(x) = \begin{cases} \infty, x \le 0; x \ge a; \\ -U_0, \ x \le b; \\ 0, b < x < a. \end{cases}$

45. Find bound states and wave functions (WF) for in $E > 0$ in $U(x) = \begin{cases} \infty, x \le 0; x \ge a; \\ -U_0, \ x \le b; \\ 0, b < x < a. \end{cases}$