LECTURE COURSE: QUANTUM MECHANICS

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Lectures, practical exercises, home tasks (problems to solve), and seminars (216 hours total)

COURSE DESCRIPTION

Aims The main goal of the course is to introduce students to quantum description of the microparticles, to present the basic ideas and the principles of quantum mechanics, their application to the description of microscopic systems. Students will study the methods of nonrelativistic and relativistic quantum theory, learn to solve the Schrödinger equation, obtain skills in the practical application of the principles of quantum mechanics.

Intended learning outcomes

At the end of the course the students should be able

- to choose and apply the problem-solving technique that is appropriate to a particular problem;
- to apply principles of quantum mechanics to calculate observables on known wave functions;
- to solve time-dependent and time-independent Schrödinger equation for different potentials;
- to understand the relation between conservation laws and symmetries;
- to apply variational method, time-independent perturbation theory and time-dependent perturbation theory to solve simple problems;
- to use angular momenta algebra and representation, raising and lowering operators, spin matrices, Pauli matrices, spin-statistics of bosons and fermions;
- be familiar with the relationships between Quantum mechanics and Electromagnetism, such as Landau Energy levels, the Aharanov-Bohm effect.

Short Course Catalogue for "Quantum Mechanics"

1. Principles of Quantum Mechanics
2. Schrödinger's equation
3. Mathematical formalism of quantum mechanics

4. Motion in a Central Field
5. The Quasi-Classical Approximation
6. Perturbation Theory
7. The Variational Principle
8. Matrix Form of Quantum Mechanics
9. Spin and the Identical Particles
10. Electronic Structure of the Atom
11. Motion in a Magnetic Field
12. Elastic Scattering
13. The method of second quantization
14. Interaction of light with matter
15. Relativistic Quantum Mechanics
16. Entangled states

Course content

1. Principles of Quantum Mechanics

1.1 Effects that require a quantum-mechanical description.

1.2. Wave properties of particles. De-Broglie hypothesis. The principle of complementarity.

1.3. The wave function. The Probability amplitude.

1.4. The principle of superposition. The expansion of the wave function in plane waves.

1.5. The uncertainty relation. Bohr's correspondence principle.

1.6. The principle of causality in quantum mechanics.

2. Schrödinger Equation

2.1. Schrödinger wave equation.

2.2. The probability density flux.

2.3. A particle in a one-dimensional rectangular potential well.

2.4. A one-dimensional oscillator.

2.5. Reflection and transmission through the rectangular potential barrier.

2.6. Singular potentials. Dirac comb (the Kronig-Penney model).

3. Mathematical Formalism of Quantum Mechanics

3.1. Linear operators. Eigenvectors and eigenvalues of the operators.

3.2. Hermitian operators. Orthogonality and normalization of the eigenvectors of Hermitian operators.

3.3. Quantum-mechanical quantities and operators. The Hilbert space of wave functions.

3.4. Dirac's "Bra" and "Ket" notations for vectors. Foundations of the representation theory in quantum mechanics.

3.5. Transition to the momentum representation. Uniformly accelerated motion.

3.6. Wigner distribution function.

3.7. Wave function and the probability of the measurement results. The average values of physical quantities.
3.8. Commutation of operators. The Heisenberg inequality. Poisson brackets in quantum mechanics.
3.9. Eigenvalues and eigenfunctions of the position and the momentum operators.
3:10. Hamiltonian operator. The stationary states of quantum systems.
3.11. Operators, eigenvalues and eigenfunctions of the angular momentum and of the square of the angular momentum.
3.12. The parity of states.
3.13. Differentiation of operators in time. The integrals of motion. A complete set of physical quantities.
3.14. The uncertainty relation for time and energy.

4. Motion in a Central Field
4.1. The wave function of a particle in a central field. Separation of variables in the Schrödinger equation.
4.2. Motion in the Coulomb field. The accidental degeneracy of the Hydrogen atom.

5. Quasi-Classical Approximation
5.1. The classical limit.
5.2. Movement in the potential well in the quasi-classical approximation. Bohr-Sommerfeld quantization rules.
5.3. Transmission through the potential barrier. The tunneling effect.
5.4. The use of quantum tunneling. Field emission, α-decay of heavy nuclei, fusion of light nuclei.

6. Perturbation Theory
6.1. Time-independent perturbation theory for a non-degenerate state.
6.2. Time-independent perturbation theory for degenerate states.
6.3. Time-dependent perturbation theory. Transition probabilities.

7. The Variational Principle

8. Matrix Form of Quantum Mechanics
8.1. The eigenfunctions and eigenvalues, given in matrix form.
8.2. Schrödinger and Heisenberg Representations.
8.3. Getting the harmonic oscillator spectrum by matrix method.

9. Spin and the Identical Particles
9.1. The spin of elementary particles. Spin operators.
9.2. The eigenfunctions and eigenvalues of the spin operators.
9.3. The total angular momentum operator. Addition of angular momenta.
9.4. Pauli equation. The spin magnetic moment.

9.5. The indistinguishability principle of the identical particles. Symmetric and antisymmetric states.
9.6. The wave functions of boson and fermion systems. The Pauli exclusion principle.
9.7. Non-interacting electrons in a three-dimensional potential well. The electron contribution to the specific heat of metals.
9.8. Consequences of the Pauli principle: the atomic nucleus; white dwarfs and neutron stars.
9.9. The wave function of the two-particle system of fermions. The exchange interaction.

10. Electronic Structure of the Atom
10.1. The wave function of a many-electron atom. Hartree-Fock method.
10.2. The statistical model of the atom.
10.3. The quantum numbers characterizing the electron state in the atom. Atomic terms.
10.4. Periodic system of elements.
10.5. Zeeman and Paschen-Back Effects. Diamagnetism of atoms.
10.6. Stark Effect.

11. Motion in a Magnetic Field
11.1. The energy spectrum and the wave function of an electron in a constant and uniform magnetic field. Landau levels.

12. Elastic Scattering
12.1. The amplitude and the scattering cross section.
12.2. The Green function of the scattering problem.
12.3. Born approximation.
11.4. Scattering in the Coulomb field. Rutherford's formula.

13. Second Quantization Method. The interaction of light with matter
13.1. The factorization method for solving the Schrödinger equation. Creation and annihilation operators.
13.2. Second quantization for bosons and fermions systems.
13.3. The quantization of the radiation field.
13.4. Coherent states of electromagnetic field.
13.5. An electron interaction with radiation. Absorption and emission of light.
13.6. The dipole transitions in atomic systems. The selection rules.
13.7. The scattering of light by atoms.

14. Quantum Mechanical Effects
14.1. Aharonov-Bohm effect
14.2. Quantum Hall effect.

15. Relativistic Quantum Mechanics
15.1. The wave equation for a relativistic particle with zero spin.
15.2. The Dirac equation.
15.3. The solution of the Dirac equation for a free particle.
15.4. The concept of a positron.
15.5. The spin of a particle in the Dirac theory. The transition to the quasi-classical Pauli equation.
16. Entangled states
16.1. Entangled states. Einstein-Podolsky-Rosen paradox.
16.2. Bell's inequalities.
16.3. Quantum cryptography. Quantum theorem about the impossibility of cloning.
16.4. Quantum computers. Shour's algorithm.

Text books

V.G.Levich, V.A.Miamlin, Yu.A.Vdovin. Theoretical Physics: An Advanced Text, Vol. 3: Quantum Mechanics (Amsterdam; London: North-Holland Publishing Co. 1973, ISBN-13: 978-0-7204-0179-0) 621 pp.

L.D. Landau and E.M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory, Vol. 3 of Course of Theoretical Physics.* E-version is available at https://ia800303.us.archive.org/27/items/QuantumMechanics_104/LandauLifshitz-QuantumMechanics.pdf

J.-L. Basdevant, J. Dalibard. *Quantum Mechanics* (Springer-Verlag, Berlin, 2002, ISBN-13: 978-3540277064) 512 pp.

Griffiths, David J. *Introduction to Quantum Mechanics*. 2nd ed. (Upper Saddle River, NJ: Pearson Prentice Hall, 2004, ISBN: 9780131118928) 480 pp.

Cohen-Tannoudji, Claude. *Quantum Mechanics*. 2 vols. (New York, NY: Wiley, 1977. ISBN: 9780471164326) 898 pp.

Sakurai, J. J. *Modern Quantum Mechanics* MA: Addison-Wesley Pub., 1994. ISBN: 9780201539295. Shankar, Ramamurti. *Principles of Quantum Mechanics*. 2nd ed. (New York, NY: Plenum Press, 1994. ISBN: 9780306447907) 676 pp.

Paul Dirac The principles of quantum mechanics

E. Merzbacher, Quantum Mechanics.

A. Messiah, Quantum Mechanics.

Web resources

http://physics.karazin.ua/en/pers_pages/yampolskii_v_v.html

http://kaf-theor-phys.univer.kharkov.ua/presentations/QM_TM_demonstration.ppsx