Quantum mechanics. Department of Physics, 6th semester.

Lesson №1. ***Mathematical tool of quantum mechanics: linear algebra, theory of linear space, theory of linear operators.***

1. Definition of linear (affine) space. Some examples of linear spaces.

2. Linear combination of vectors. Linear dependence and linear independence of vectors. Dimension of linear space. Basis vectors.

3. Euclid spaces. Dot product. Normalizing and orthogonality of vectors. Orthonormal basis. Examples of dot products.

4. Hilbert space. Space L^2. Concept of full system of functions for infinite-dimensional spaces.

5. Definition of liner operator $\hat{L}\left(\alpha f + \beta g\right) = \alpha \hat{L} f + \beta \hat{L} g$.

<u>Task 1.</u> Verify linearity of the following operators (from *НКК № 1.1*):

a) $\hat{I}\psi(x) = \psi(-x)$ –inversion operator;

b) $\hat{T}_a\psi(x) = \psi(x+a)$ – shift operator;

c) $\hat{M}_c\psi(x) = \sqrt{C}\psi(Cx), \quad C > 0$ – operator of scale conversion;

d) $\hat{K}\psi(x) = \psi^*(x)$ – operator of complex conjunction;

e) $\hat{P}_{12}\psi(x_1, x_2) = \psi(x_2, x_1)$ – permutation operator.

6. Ways linear operators to define: correspondence rule, integral form, matrix form.

7. Operator operations.

7.1. Unity operator, null operator.

7.2. Sum of operators.

7.3 Product of operators.

7.4. Commutator of two operators.

7.5. Inverse operator. Normal operator.

<u>Task 2.</u> Find an operator inverse to product of operators $\hat{A}$ and $\hat{B}, \left(\hat{A}\hat{B}\right)^{-1} - ?$

7.6. Definition of Hermitian conjugated operator.

<u>Tasks 3-5.</u> Verify, that $(\hat{L}^\dagger)^\dagger = \hat{L}; \quad (\alpha\hat{L})^\dagger = \alpha^*\hat{L}^\dagger; \quad (\hat{A}\hat{B})^\dagger = \hat{B}^\dagger\hat{A}^\dagger$.

7.7. Definition of Hermitian (self-adjoint) operator.

7.8. Definition of unitary operator.

8. Eigenequation. Eigenfunctions and Eigenvalues:

$$\hat{A}\psi = \lambda \psi$$

8.1. Properties of Eigenfunctions and Eigenvalues of Hermitian operator.

8.2. Properties of eEigenvalues of unitary operator.

Task 6. Expand the brackets in operator expression $\left(x + \frac{d}{dx} \right)^2$ (*Hr. №8(a)*)

Home tasks: *ГКК 1.1-1.10, Hr. №8, №9*

1. For every operator from *НКК 1.1* (see **task 1**) find Hermitian conjugated and inverse operators.

2. Find operators which are Hermitian conjugated to operators:

a) $\frac{d}{dx}, \quad i\frac{d}{dx}, \quad -\infty < x < \infty$; b) $i\frac{\partial}{\partial r}, \quad 0 \le r < \infty$ (*НКК 1.2*)

3. Prove that the following operators are Hermitian:

a) $\hat{L}^{\dagger}\hat{L}, \quad \hat{L}\hat{L}^{\dagger}, \quad \hat{L} + \hat{L}^{\dagger}, \quad i\left(\hat{L} - \hat{L}^{\dagger}\right)$ (*НКК 1.3*)

b) $\hat{A}\hat{B} + \hat{B}\hat{A}, \quad i\left(\hat{A}\hat{B} - \hat{B}\hat{A}\right)$, if $\hat{A}$ и $\hat{B}$ – Hermitian operators. (*НКК 1.6*)

4. Verify, that if $\hat{C}$ – Hermitian operator, than operator $\hat{G} = \hat{A}\hat{C}\hat{A}^{\dagger}$ is also Hermitian. (*НКК 1.4*)

5. Verify, that arbitrary operator can be presented as $\hat{F} = \hat{A} + i\hat{B}$, where $\hat{A}$ and $\hat{B}$ – Hermitian operators. (*НКК 1.5*)

6. Operator $\hat{F}$ is not Hermitian. In which case operator $\hat{F}^2$ is Hermitian? (*ГКК 1.7*)

7. Verify, that algebraic manipulations with commutators hold for distributive property, namely that sum commutator equals to sum of $\hat{B}\left[\hat{A},\hat{C}\right]$ (*НКК 1.8*)

$$\left[\sum_i \hat{A}_i, \sum_k \hat{B}_k \right] = \sum_{i,k} \left[\hat{A}_i, \hat{B}_k \right].$$

8. Verify, that

$$\left[\hat{A}\hat{B},\hat{C}\right] = \hat{A}\left[\hat{B},\hat{C}\right] + \left[\hat{A},\hat{C}\right]\hat{B};$$

$$\left[\hat{A},\hat{B}\hat{C}\right] = \hat{B}\left[\hat{A},\hat{C}\right] + \left[\hat{A},\hat{B}\right]\hat{C};$$ (*ГКК 1.9*)

9. Prove the Jacobi identity for operator's $\hat{A}, \hat{B}, \hat{C}$ commutators

$$\left[\hat{A},\left[\hat{B},\hat{C}\right]\right]+\left[\hat{B},\left[\hat{C},\hat{A}\right]\right]+\left[\hat{C},\left[\hat{A},\hat{B}\right]\right]=0.$$

10. Expand the brackets in operator expression (*Hr. № 8(b-e)*):

б) $\left(\frac{d}{dx}+\frac{1}{x}\right)^3$; в) $\left(x\frac{d}{dx}\right)^2$; г) $\left(\frac{d}{dx}x\right)^2$; д) $\left[i\hbar\nabla+\vec{A}(\vec{r})\right]^2$;

е) $\left(\hat{L}-\hat{M}\right)\left(\hat{L}+\hat{M}\right)$.

11. Find commutators of operators:

а) x and $\frac{d}{dx}$; б) $i\hbar\nabla$ and $\vec{A}(\vec{r})$; в) $\frac{\partial}{\partial\varphi}$ and $f(r,\theta,\varphi)$. (*Hr .№ 9)*)

HKK- Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981
Hr. - Hrechko, Suhakov, Tomasevich, Fedorchenko Collection of theoretical physics problems, 1984