Quantum mechanics. Department of physics. 7th semester.

Lesson №16. ***Spin.***

1. Checking hometask. Evaluate in quasi-classical approach the transmission coefficient of a rectangular barrier

$$U(x)=\begin{cases}0, & x<0;\, x>a;\\ U_0, & 0<x<a.\end{cases}$$

Specify the criterion of applicability of the obtained result.

2. In quantum mechanics an elementary particle is attributed to some own momentum that is not associated with its motion in space. Own mechanical momentum is called spin. Spin operator $\hat{\vec{S}}$ has general properties of quantum mechanical momentum, which means it satisfies same commutation relations, as orbital angular momentum operator $\hat{\vec{l}}$:

$$[\hat{S}_i,\hat{S}_j]=i\varepsilon_{ijk}\hat{S}_k;\quad [\hat{\vec{S}}^2,S_j]=0;\quad i,j,k=x,y,z,$$

ε_{ijk} – Levi-Civita symbol. There are particles with integer (bosons) and half-integer (fermions) spin.

$$\text{СЗ } \hat{\vec{S}}^2 \quad \lambda_{S^2}=S(S+1);$$

$$\text{СЗ } \hat{S}_z \quad \lambda_{S_z}=S_z=-S,-S+1,\dots,S-1,S \quad (2S+1 \text{ total values})$$

$$S=0,\frac{1}{2},1,\frac{3}{2},2,\dots .$$

Full wave function of particle with spin $\Psi(\vec{r},\sigma,t)$ depends on discrete spin variable σ. As spin variable one can choose z-projection S_z value of spin. Wave function of the particle with spin S has $2S+1$ components.

2.1. Spin operator can be presented in matrix form. For spin $S=1/2$ it is the two-row Pauli matrix

$$\hat{\vec{S}}=\frac{1}{2}\hat{\vec{\sigma}},$$

$$\hat{\vec{\sigma}}=\left(\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z\right);\ \hat{\sigma}_x=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \hat{\sigma}_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\ \hat{\sigma}_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

Remind, that Pauli matrices properties were investigated in lessons № 3 and № 4 during last semester.

Task 1. For the particle with spin $S = 1/2$ find eigenfunctions and eigenvalues of spin operators $\hat{S}_j$ $j = x, y, z$. (НКК № 5.1)

Task 2. Prove formulas

$$\left(\vec{a}\cdot\hat{\vec{S}}\right)^2 = \frac{a^2}{4}; \quad \left(\hat{\vec{S}}\cdot\vec{a}\right)\left(\hat{\vec{S}}\cdot\vec{b}\right) = \frac{1}{4}\vec{a}\cdot\vec{b} + \frac{i}{2}\hat{\vec{S}}\cdot\left(\vec{a}\times\vec{b}\right).$$

3. Test (~ 20 minutes). Test contains 2 tasks, highest mark – **5 points**.

Hometask НКК 5.2- 5.4.

LL – Landau LD, Lifshits IM, Quantum Mechanics
НКК- Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981
Flugge Z. Problems in quantum mechanics. Part 1, Part 2.1974