Quantum mechanics. Department of physics. 7th semester.

Lesson №10. ***Movement in central field: the two-body problem in quantum mechanics, flat rotator, spatial rotator.***

1. Homework check.

Tasks 1-2. Calculate commutators $\left[L_x^2, y^2\right]$, $\left[l_+^2, l_-\right]$

Задача 3. Find average value $\left\langle \hat{l}_x \hat{l}_z \right\rangle$ in state ψ_m with certain value of momentum z-projection m.

2. Two-body problem

$$\hat{H} = \frac{\hat{\vec{p}}_1^{\,2}}{2m_1} + \frac{\hat{\vec{p}}_2^{\,2}}{2m_2} + U\left(|\hat{\vec{r}}_1 - \hat{\vec{r}}_2|\right)$$

in quantum mechanics, as in classic mechanics, is brought to the problem of movement in central field

$$\hat{H} = -\frac{\hbar^2}{2(m_1+m_2)}\Delta_R - \frac{\hbar^2}{2\mu}\Delta_r + U(r), \text{ where}$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$ – position vector of two particles' center of inertia

$$\vec{r} = \vec{r}_1 - \vec{r}$$ – position vector of two particles' relative motion.

$$\mu = \frac{m_1 m_2}{m_1+m_2}$$ – reduced mass of two particles.

$$\psi(\vec{r}_1, \vec{r}_2) \to \psi(\vec{R}, \vec{r}) = \psi(\vec{R})\psi(\vec{r}), \quad \psi(\vec{R}) = \exp(i\vec{K}\vec{R}).$$

2.1. Hamiltonian of the particle, which moves in the central field is

$$\hat{H} = -\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\Delta_{\theta\varphi}\right] + U(r) = -\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\hbar^2\hat{\vec{l}}^{\,2}}{2\mu r^2} + U(r);$$

$$\Delta_{\theta\varphi} = \left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right] = -\hat{\vec{l}}^{\,2}.$$

Commutation relations for $\hat{H}, \hat{\vec{l}}^{\,2}, \hat{l}_z$ are

$$\left[\hat{H},\hat{\vec{l}}^{\,2}\right]=0,\quad \left[\hat{H},\hat{l}_z\right]=0,\quad \left[\hat{\vec{l}}^{\,2},\hat{l}_z\right]=0.$$

Separation of variables: $\psi(\vec{r})=\psi(r,\theta,\varphi)=R(r)Y_{lm}(\theta,\varphi)$.

2.2. Equation for radial part of wave function $R(r)$

$$-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+U_{eff.}(r)R(r)=ER(r);$$

$$U_{eff.}=U(r)+\frac{\hbar^2 l(l+1)}{2\mu r^2}.$$

Task 4. Find energy levels and normalized wave functions of rigid plane rotator with Hamiltonian $\hat{H}=\frac{\hbar^2\hat{l}_z^2}{2I},\quad I=\mu a^2$.(НКК № 4.1)

Task 5. Find energy levels and normalized wave functions of rigid spatial rotator with Hamiltonian $\hat{H}=\frac{\hbar^2\hat{\vec{l}}^{\,2}}{2I},\quad I=\mu a^2$.(НКК № 4.3)

Hometask НКК № 3.37, 4.28, 4.33.

НКК- Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981
Hr. - Hrechko, Suhakov, Tomasevich, Fedorchenko Collection of theoretical physics problems, 1984